Chapter 8: Data-Driven Models

8.2 Function Tutorial

Background: Data-Driven Models

- So far we've been given a model ($dP/dt = kP$) and used it to generate data.
- Research often requires us to build a model based on (limited) data.
- To do this, it helps to be familiar with the standard mathematical functions used for building models.

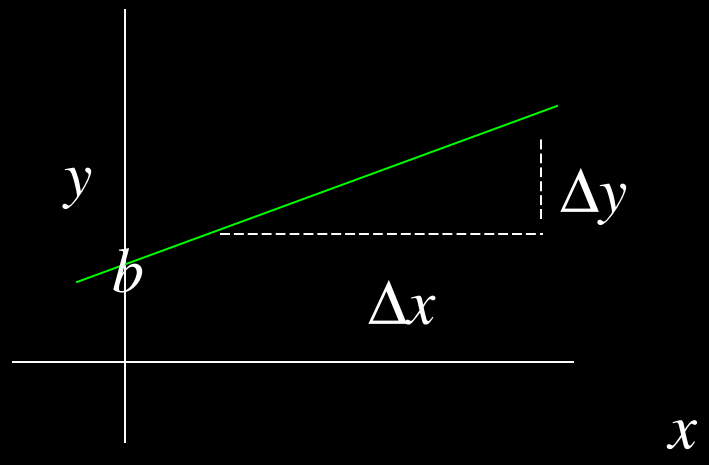
Linear Functions

- Straight line is the simplest model
- Human beings are biased toward viewing patterns as straight lines with positive slope (Busemeyer *et al.* 1997)

Linear Functions

$$y = mx + b$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

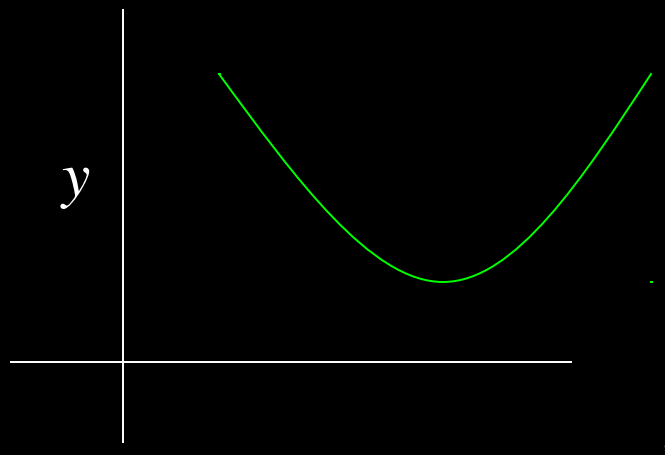


m: slope

b: intercept

Quadratic Functions

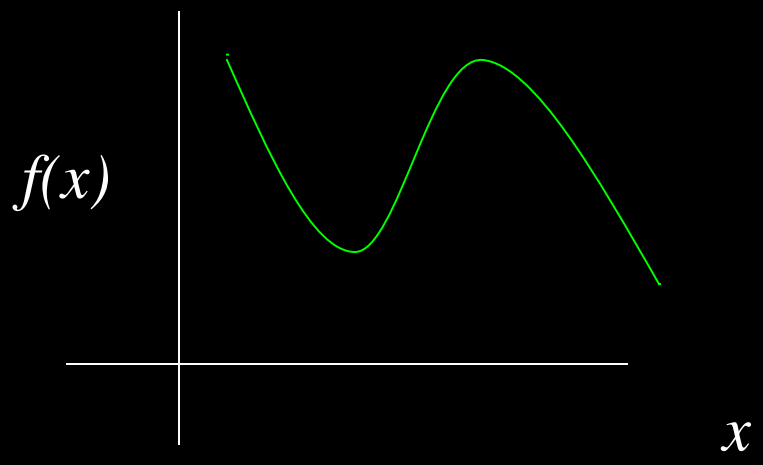
$$f(x) = a_2 x^2 + a_1 x + a_0$$



a special case of …

Polynomial Functions

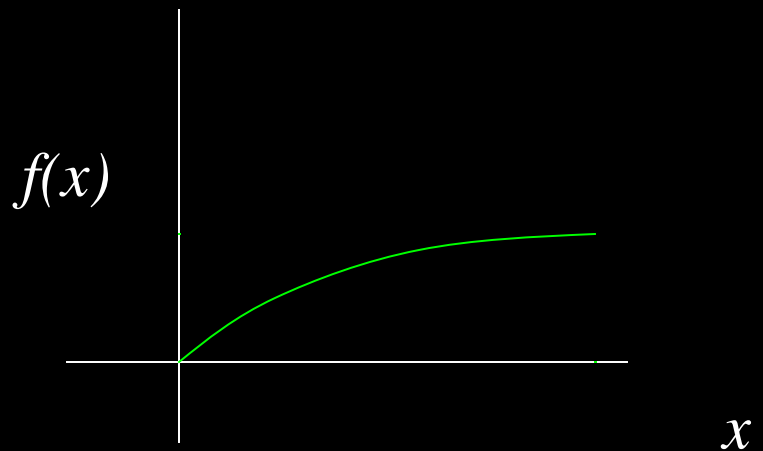
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$



$$f(x) = \sum_{i=0}^{n} a_i x^i$$

n: degree

Square Root Function

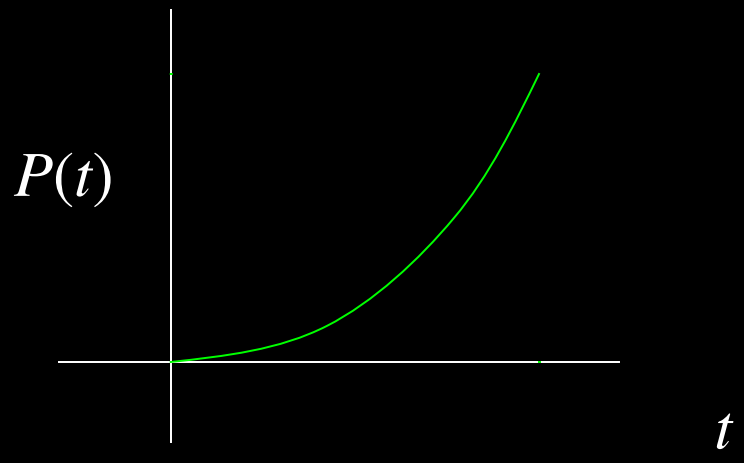


Useful when quantities are already squared: *e.g.* distance between two points (x_1, y_1) and (x_2, y_2) =

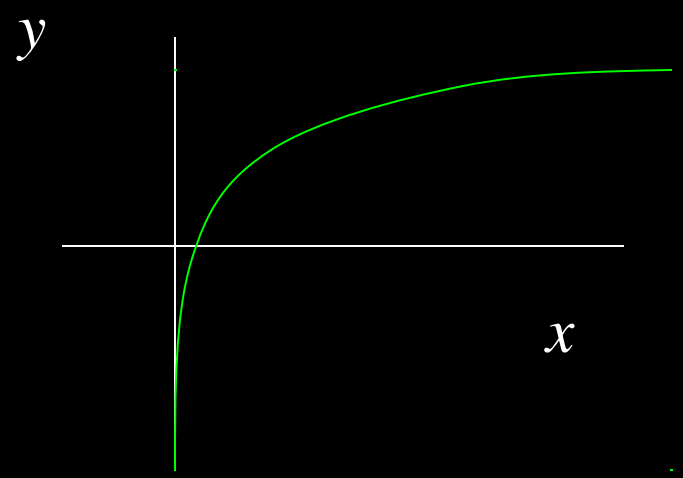
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Exponential Function

$$P(t) = P_0 e^{rt}$$

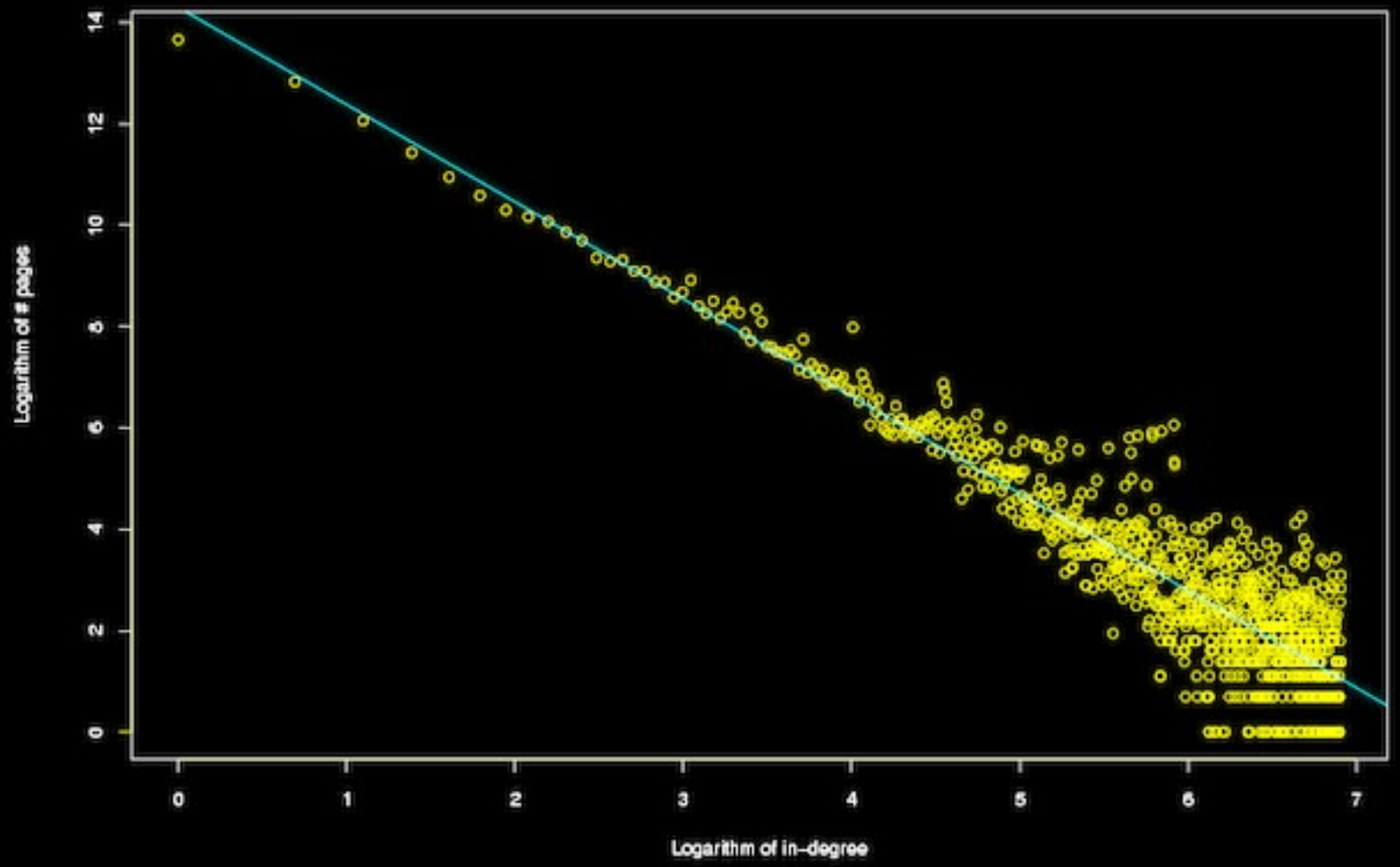


Logarithmic Function



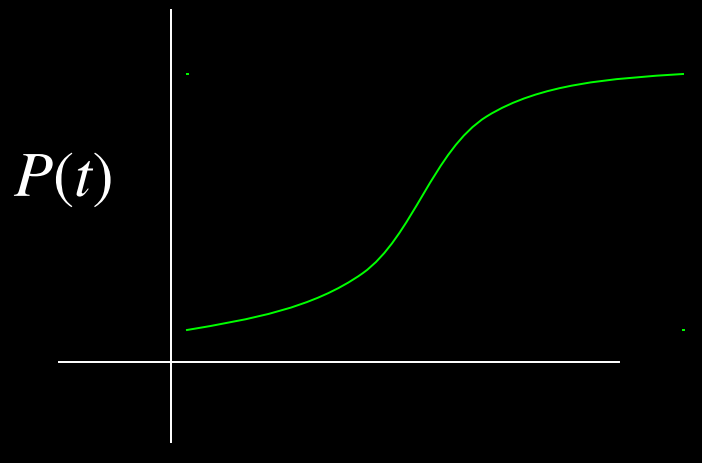
Useful when dealing with inherently exponential measures, *e.g.* Richter scale for earthquakes.

Log/Log Plot for Power Laws



Logistic Function

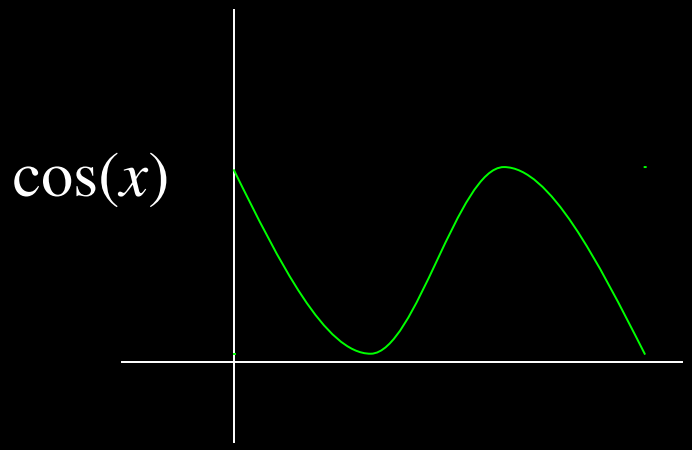
$$\frac{dP}{dt} = r\left(1 - \frac{P}{M}\right)P$$



$$P(t) = \frac{MP_0}{(M - P_0)e^{-rt} + P_0}$$

Trigonometric Functions

- Useful for for modeling cyclic processes (ocean temperature, blood sugar level)



- Because $\sin(\theta) = \cos(\theta - \pi/2)$, we generally only need cos.