8.3 Empirical Models

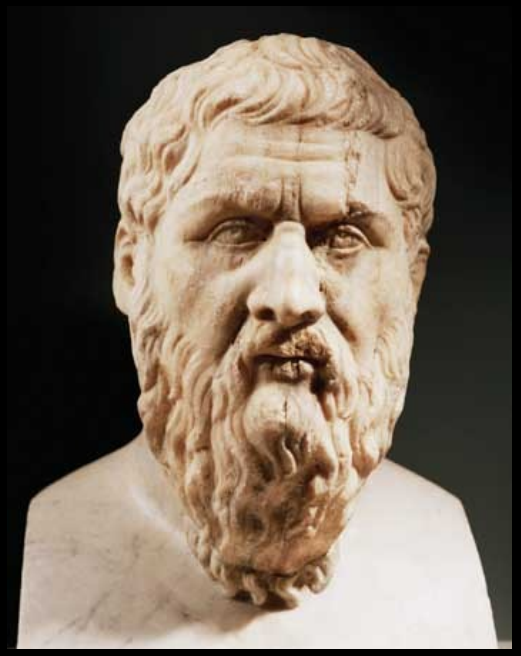
Definition

An empirical model is based only on data and is used to predict, not explain, a system. An empirical model consists of a function that captures the trend of the data.

Empiricism *vs*. Rationalism

Aristotle

(384-322 BC)



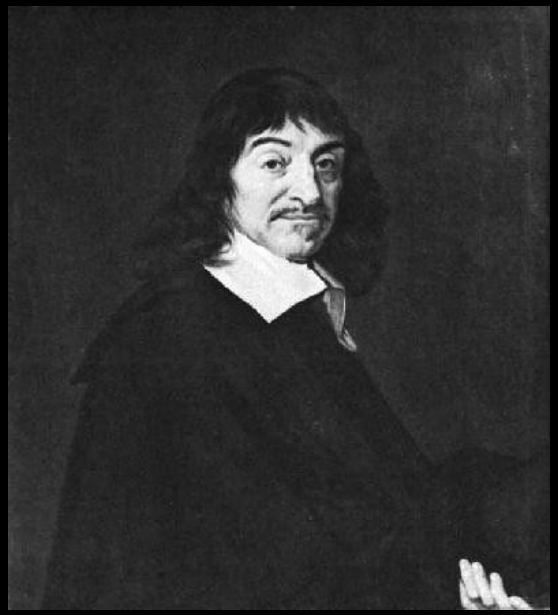
Plato

(ca.428-348 BC)

Empiricism *vs*. Rationalism

Hume

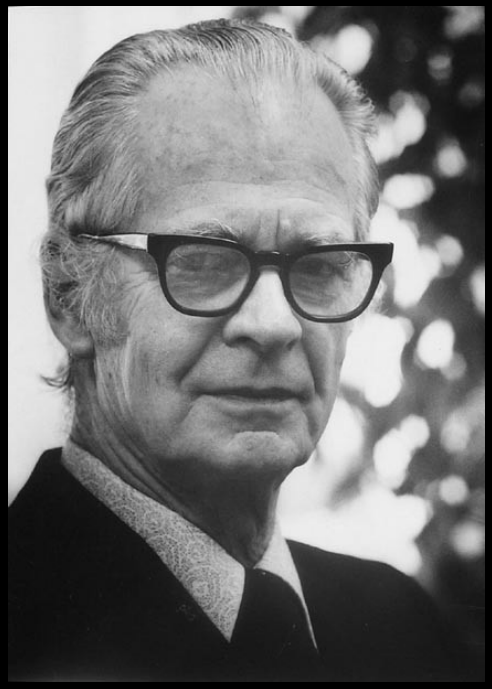
(1711-1776)



Descartes

(1596-1650)

Empiricism *vs*. Rationalism

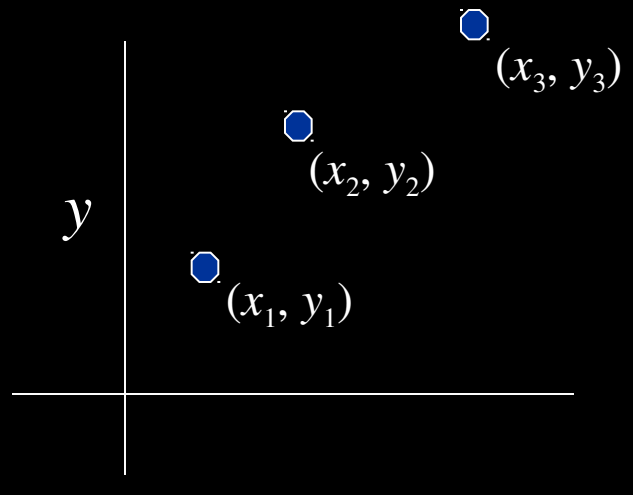


Skinner
(1904-1990)

Chomsky
(1928-)

Linear Regression

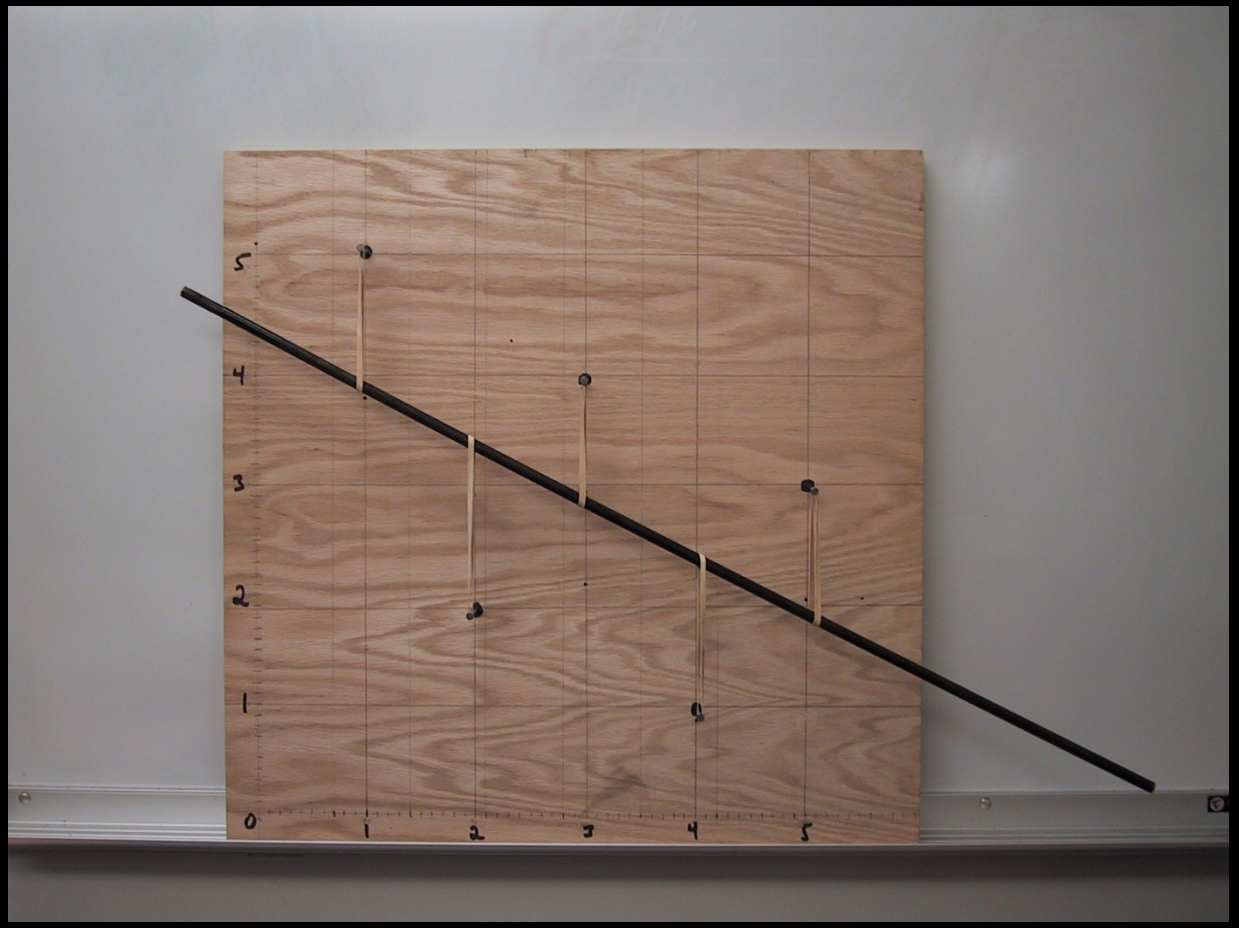
To obtain the parameters m and b in $y = ax + b$ we attempt to minimize the total difference between the model and the observed data:



$$\sum_{i=1}^{n} (mx_i + b - y_i)^2$$

(also called linear least squares)

Linear Least Squares: Analog Solution



Linear Least Squares: Numerical Solution

Calculus gives us:

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{n\sum x_i^{\,2} - (\sum x_i)^2}$$

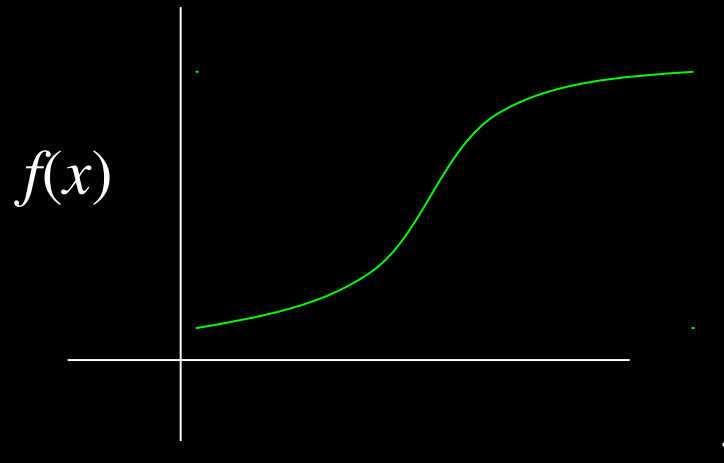
General Linear Empirical Model

$$y = a_1 x_1 + a_2 x_2 + \ldots + a_n x_n$$

- The a_i are often called the parameters of the model. The x_i can be anything we wish to combine ($y = mx + b$; $y = ax^2 + bx + c$, etc.)

- Models that can not be expressed in this way are said to be nonlinear. For example …

Logistic Regression



$$f(x) = \frac{1}{1 + e^{-(ax+b)}}$$

- Useful for modeling categorical (yes/no) decisions

- No explicit formula for solution: have to use iterative (repetitive/looping refinement) method like Levenberg-Marquardt (gradient descent)