

## 2.2.2 Abstract Syntax

- Recall BNF definition of  $\lambda$ -calculus expressions:

$\langle \text{expression} \rangle$  ::=  $\langle \text{identifier} \rangle$   
::= (**lambda** ( $\langle \text{identifier} \rangle$ )  $\langle \text{expression} \rangle$  )  
::= ( $\langle \text{expression} \rangle$   $\langle \text{expression} \rangle$  )

- Uses *concrete* syntax: includes parens, and keywords like **lambda**
- We can use this to define an *abstract* datatype without these redundant items:

**(define-datatype expression expression?**

**(var-exp ::= <identifier>  
          (id symbol?))**

**(lambda-exp ::= (lambda (<identifier>) <expression>  
                  (id symbol?)  
                  (body expression?))**

**(app-exp ::= (<expression> <expression>)  
          (rator expression?)  
          (rand expression?)))**

- We can use the abstract data type together with the BNF definition, to make an *abstract syntax tree*
- First, we annotate the BNF with tree notation:

<expression> ::= <identifier>

**var-exp (id)**

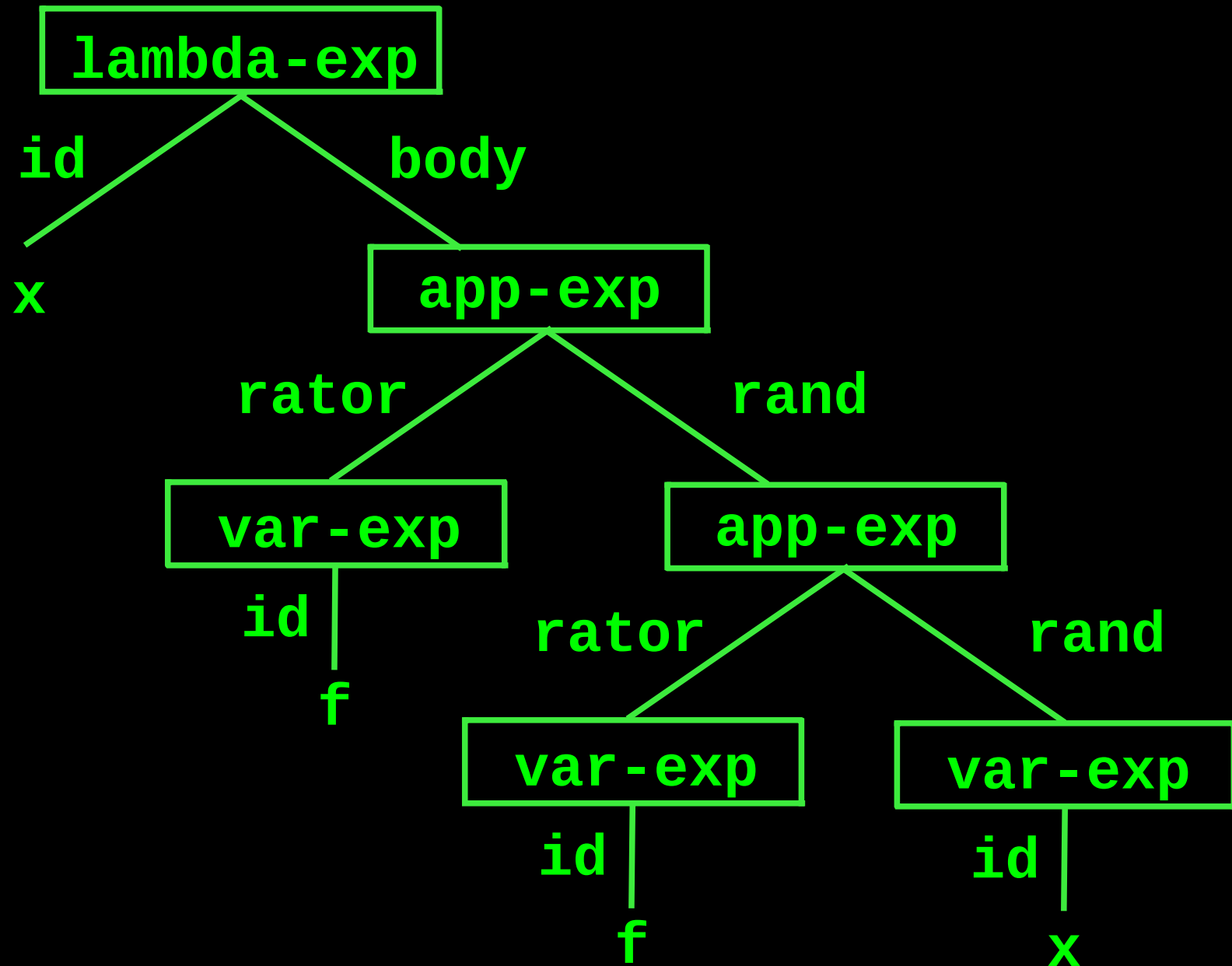
::= (**lambda** (<identifier>) <expression>)

**lambda-exp (id body)**

::= (<expression> <expression>)

**app-exp (rator rand)**

Example: (lambda (x) (f (f x)))



We can use the defined datatype to simplify functions:

```
(define occurs-free?
  (lambda (var exp)
    (cases expression exp
      (var-exp (id) (eqv? id var))
      (lambda-exp (id body)
        (and (not (eqv? id var))
              (occurs-free? var body)))
      (app-exp (rator rand)
        (or (occurs-free? var rator)
            (occurs-free? var rand))))))
```

Example:

```
> (define e (lambda-exp 'a (var-exp 'b)))  
> (expression? e)  
#t  
> (occurs-free? 'a e)  
#f  
> (occurs-free? 'b e)  
#t
```

But this is awkward. We want to *parse* actual expressions into the data structure....

```
(define parse-expression
  (lambda (e)
    (cond
      ((symbol? e) (var-exp e))
      ((pair? e)
       (if (eqv? (car e) 'lambda)
           (lambda-exp (caadr e)
                       (parse-expression (caddr e)))
           (app-exp (parse-expression (car e))
                    (parse-expression (cadr e)))))
      (else (eopl:error 'parse-expression
                          "Syntax error: ~s" e))))))
```

```
> (parse-expression '(lambda(a) (a b)))
#(struct:lambda-exp a #(struct:app-exp
                        #(struct:var-exp a) #(struct:var-exp b)))
```